# Sticky Information and the Taylor Principle

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## Motivation and Contribution

This paper presents determinacy bounds on monetary policy for the sticky information model (Mankiw and Reis, 2002)

- Determinacy bounds on monetary policy previously unobtainable
- Existing analysis relies on time domain methods that only approximate the solution
- We provide frequency domain methods that provide closed form results
- New insights into dynamics and bounds on monetary policy

⇒ These facts are at odds with standard textbook macro models!

⇒ How can we overcome these challenges?

**Our contribution:** use frequency domain methods that enable closed form and conservative restrictions on monetary policy rules in the absence of model certainty:

### Sticky Information and the Taylor Principle

The Sticky Information model in the frequency domain:

…derives a fully recursive expression of the sticky information Phillips curve

…finds determinacy bounds on monetary policy previously unobtainable

…remains analytically tractable

…gives new insights in model dynamics

## The z-transform

Consider the unilateral square-summable sequence of real numbers

$$\{c_j\}_{j=0}^{\infty}$$

where $c_j \in \mathbb{R}$ such that $\sum_{n=0}^{\infty} c_j^2 < 0$.

Then, the well-defined random variable is a function of the underlying shocks:

$$x_t = \sum_{j=0}^{\infty} c_j \epsilon_{t-j} = \sum_{j=0}^{\infty} c_j L^j \epsilon_t = c(L)\epsilon_t$$

The z-transform of the sequence $c_j$ is then given by the following function

$$x(z) = \sum_{j=0}^{\infty} c_j z^j$$

where $z \in \mathbb{C}$ and $z = e^{i\omega}$ for the angular frequency $\omega \in [-\pi, \pi]$. Scaling in the z domain connects initial conditions, unconditional moments and dampening

$$x(\lambda z) = \sum_{j=0}^{\infty} \lambda^j c_j z^j, \quad x(0) = c_0 \quad IFFT(x(z^{-1})\sigma_\epsilon^2 x(z)))_{|z|=1} = \sigma_x^2$$

For $0 < \lambda < 1$, more distant (lower frequency) movements' contribution to moments reduced

## Sticky Information versus Sticky Prices

Sticky Price Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

Sticky Information Phillips Curve:

$$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{\infty} \lambda^i E_{t-i-1}\left[\pi_t + \xi\left(y_t - y_{t-1}\right)\right]$$

$\xi > 0$ degree of strategic comp., and $0 < 1-\lambda < 1$ is the prob. of an info update. Inflation depends on output, past expectations of current inflation and output growth

Standard Dynamic IS Demand:

$$y_t = E_t y_{t+1} - \sigma R_t + \sigma E_t \pi_{t+1}$$

Monetary policy

$$R_t = \phi_\pi \pi_t + \phi_y y_t$$

The infinite regress of lagged expectations precludes a recursive representation in the time domain. Therefore we express the model in the frequency domain.

## Recursive Frequency Domain Representation

Sticky Price Phillips curve in the Frequency Domain:

$$\pi(z) = \beta\frac{1}{z}(\pi(z) - \pi_0) + \kappa y(z)$$

Sticky Information Phillips curve in the Frequency Domain:

$$y(z) = \frac{\lambda}{\xi}\frac{1}{1-\lambda z}\pi(\lambda z) + \lambda y(\lambda z)$$

### Interpretation of the Sticky Information Phillips curve

- Output gap at freq $z$ depends on output gap and inflation at dampened frequencies, $\lambda z$
- $\lambda$ introduces stickiness in the frequency domain: If the fraction of firms with an info update is low (high), output gap is driven more strongly by inflation at low (high) frequencies
- Output gap depends only on inflation at higher frequencies: vertical SIPK in the long run

## Determinacy in the Frequency Domain

Continuation of analytic function over a removable singularity pins down initial condition

$$x_t = \alpha E_t x_{t+1} + \epsilon_t \quad \overset{z}{\rightarrow} \quad (1 - z\frac{1}{\alpha})x(z) = x_0 - \frac{z}{\alpha}$$

If $|\alpha| < 1$, then for $z = \alpha$ there is a removable singularity inside the unit disk

$$\lim_{z\to\infty}(1 - z\frac{1}{\alpha})y(z) = 0$$

Sticky Price Determinacy

$$\begin{bmatrix}-\beta & 0\\ \sigma & 1\end{bmatrix}\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} = \begin{bmatrix}-1 & \kappa\\ \sigma\phi_\pi & 1+\sigma\phi_y\end{bmatrix} z \begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} + \begin{bmatrix}-\beta & 0\\ \sigma & 1\end{bmatrix}\begin{bmatrix}\pi_0\\ y_0\end{bmatrix}$$

Decoupling as follows gives two singularities $\lim_{z\to 1/\lambda_i}(1 - z\lambda_i)w_i(z) = 0, \quad i = 1,2$

$$\left[w_1(z)\ w_2(z)\right]' = V^{-1}\left[\pi(z)\ y(z)\right]'$$

- Singularities inside the unit circle if $1 - \frac{1-\beta}{\kappa}\phi_y < \phi_\pi$

⇒ Standard SP result, output gap targeting can substitute for inflation

- Taylor principle sufficient but not necessary for determinacy

Sticky Information Determinacy

$$\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} = \begin{bmatrix}\phi_\pi & \frac{1+\sigma\phi_y-\lambda}{\sigma}\\ 0 & \lambda\end{bmatrix} z \begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} + \begin{bmatrix}\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}\\ 0\end{bmatrix} y_0 + \begin{bmatrix}-\frac{\lambda}{\sigma\xi} & -\frac{\lambda}{\sigma}(1-\lambda z)\\ \frac{\lambda}{\xi} & \lambda(1-\lambda z)\end{bmatrix}\begin{bmatrix}\pi(\lambda z)\\ y(\lambda z)\end{bmatrix}$$

$x(\lambda z)$: Irrelevant for determinacy (a property of $x(z)|_{|z|=1}$)

- Backward looking (recursive in frequency) SI PK gives one initial condition $\xi\frac{1-\lambda}{\lambda}y_0 = \pi_0$
- Of the two eigenvalues ($\lambda$ and $\phi_\pi$, one and only one must provide a removable singularity to pin down remaining condition
  - As the probability of no information update, $0 < \lambda < 1$, certainly inside the unit circle
- Hence it must hold that $1 < \phi_\pi$

⇒ A stricter bound, output gap targeting cannot substitute for inflation

- Taylor principle sufficient and necessary for determinacy

## Extension

Consider the more general Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\phi_\pi(\alpha_\pi \pi_t + (1-\alpha_\pi)E_t\pi_{t+1}) + \phi_y(\alpha_y y_t + (1-\alpha_y)\Delta y_t)\right]$$

- $0 \le \rho_R < 1$ allows for interest rate smoothing
- $\alpha_\pi$: both contemporaneous ($\alpha_\pi = 1$) as well as future ($\alpha_\pi = 0$) inflation targeting
- $\alpha_y$: output gap level ($\alpha_y = 1$) as well as output gap growth ($\alpha_y = 0$) targeting

The IS and Taylor rule in frequency representation

$$\begin{aligned}&\left[1 - (1-\rho_R)\phi_\pi(1-\alpha_\pi) - z(\rho_R(1-\phi_\pi\alpha_\pi) + \phi_\pi\alpha_\pi)\right]\pi(z)\\ &= \left[1 - (1-\rho_R)\phi_\pi(1-\alpha_\pi) - z\rho_R\right]\pi(0) - \frac{1-z\rho_R}{\sigma}y(0)\\ &+ \left[\frac{1}{\sigma} + \phi_y(1-\rho_R) - z\left(\frac{1+\rho_R}{\sigma} + \phi_y(1-\rho_R)(1-\alpha_y)\right) + z^2\frac{\rho}{\sigma}\right]y(z)\end{aligned}$$

This posits a relation between $\pi(z)$ and $y(z)$

Sticky Price Determinacy Sticky Price Phillips curve

$$(1 - \beta\frac{1}{z}\pi(z) = -\beta\frac{1}{z}\pi_0 + \kappa y(z)$$

- posits a long-run ($|z| = 1$) tradeoff between inflation and the output gap
- fragile, specification specific determinacy bounds
- Taylor principle not directly relevant

Sticky Information Determinacy

Sticky Information Phillips curve

$$y(z) = \frac{\lambda}{\xi}\frac{1}{1-\lambda z}\pi(\lambda z) + \lambda y(\lambda z) = \frac{1}{\xi}\sum_{j=1}^{\infty}\frac{\lambda^j}{1-\lambda^j z}\pi(\lambda^j z)$$

- no long-run ($|z| = 1$) tradeoff between inflation and the output gap
- Backward looking (recursive in frequency) SI PK gives one initial condition $\xi\frac{1-\lambda}{\lambda}y_0 = \pi_0$
- Removable singularity must be in the IS + Taylor rule wrt inflation

$$\lim_{z\to\omega}\left[1 - (1-\rho_R)\phi_\pi(1-\alpha_\pi) - z(\rho_R(1-\phi_\pi\alpha_\pi) + \phi_\pi\alpha_\pi)\right]\pi(z) = 0$$

- Is relevant for determinacy if and only if

$$|\omega| = |\frac{1 - (1-\rho_R)\phi_\pi(1-\alpha_\pi)}{\rho_R(1-\phi_\pi\alpha_\pi) + \phi_\pi\alpha_\pi}| < 1$$

- No amount or type of output targeting can replace necessary focus on inflation!

## Sticky Information in the Frequency Domain

Sticky Information + z-transform

- Sticky Information Phillips Curve:
  - presents a recursive representation in the time domain (infinite regress)
- z-transform: transforms the model into the frequency domain
  - allows for a closed form solution

Sticky Information in the Frequency Domain: recursive representation of infinite regress ✓ closed form solution ✓ ⇒ new insights into monetary policy!